

Disappearance of fractional statistics in Schrieffer-Wilczek theory.

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The paper of Arovas, Schrieffer and Wilczek is corrected. It is found that the Berry's phase is vanishingly small. Accordingly, the statistical vector potential also becomes negligibly small. The need for intermediate statistics is then obviated.

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1. Introduction

As the magnetic field is varied, the Hall conductance, $\sigma_{xy} = I_x/E_y = \nu e^2/h$ shows plateaus at $\nu = n/m$ where n and m are integers, with m being odd. Laughlin¹ has shown that at the plateaus, the quasiparticle charge in the “incompressible fluid” is $\pm e^* = e/m$. Arovas et al² make an effort to determine the statistics of the quasiparticles of fractional charge by determining the Berry’s phase in the “incompressible fluid”.

In this letter, we report that the calculation of the phase for incompressible fluid has not been carried out correctly by Arovas et al² and if performed correctly will not give the same result. In particular, the Berry’s phase found by Arovas et al is large but the correct value is vanishingly small. Similarly, the correction to the vector potential found by Arovas et al² is large but when calculated properly, becomes negligibly small. Therefore, the need for intermediate statistics is completely obviated.

2. Theory

Arovas et al report that the quasiparticle charge is $\pm e^* = \pm e/m$. Actually, Laughlin found that the charge density is $\sigma_m = 1/[m(2\pi a_o^2)]$ where $a_o^2 = \hbar c/eB$. On the basis of this result Laughlin introduced incompressibility so that a_o can be left out as a fixed number and the charge is determined to be e/m . This result is in error because there are at least three possibilities. (i) Keep the charge at e/m and a_o^2 as a constant, (ii) fix the charge at e and change the area from a_o^2 to ma_o^2 and (iii) the charge is e , the area is a_o^2 and m is kept as a number. Laughlin has chosen only the first of these possibilities. Hence the question is that what scientific basis has been used to pick up (i) and leave out (ii) and (iii)? The question is about the selection of (i) by ignoring (ii) and (iii). There is obviously no reason to prefer (i) and there is no method which dictates that only (i) is the correct answer. In fact, a linear combination of all the three should be chosen if the correct answer is to be obtained. In a product of e/ma_o^2 it is not correct to say that the charge e has been changed to e/m because it is also possible that e is unchanged and only a_o^2 is changed to ma_o^2 . Laughlin’s choice is therefore arbitrary. We wish to emphasize that there are other solutions which were left out by Laughlin. Arovas et al take $\pm e^* = \pm e/m$

but obviously the correct result is not the charge e^* but the charge density, $e/2m\pi a_o^2$ which is the charge density per unit area. It is possible to consider the one dimensional result, e/ma_o , which is the charge per unit length or the three dimensional result, i.e., the charge per unit volume, $e/[m(4\pi a_o^3/3)]$. We shall consider only the charge per unit area. Once the result for the area is known, the length or the volume results can be trivially derived.

Arovas et al calculate the change of phase, γ of a state having a quasihole localized at z_o . As z_o adiabatically moves around a circle of radius R enclosing flux ϕ , the change in phase is,

$$\frac{e^*}{\hbar c} \oint \vec{A} \cdot d\vec{l} = 2\pi \left(\frac{e^*}{e}\right) \left(\frac{\phi}{\phi_o}\right). \quad (1)$$

This is the phase gained by the quasiparticle of charge e^* in going around the loop. Actually, the Laughlin calculation is concerned with the charge per unit area and not only the quasiparticle charge, so that e^* should be replaced by e^*/ma_o^2 . **Arovas et al have left out the factor ma_o^2 by using the argument of incompressibility but a_o involves the magnetic field which causes field dependence.** Since, e^* is the charge in a magnetic field, it uses $a_o^2 = \hbar c / eB$ to determine the relevant area. Therefore, the charge e in the absence of a magnetic field is linked to the Bohr radius, a_B . The corrected form of the phase must have ma_o^2 . Since the phase must be dimensionless, the corrected form of the phase should be,

$$\gamma = 2\pi \left(\frac{e^*}{e}\right) \left(\frac{a_B^2}{ma_o^2}\right) \left(\frac{\phi}{\phi_o}\right). \quad (2)$$

This means that the phase factor is linearly dependent on the magnetic field. At the fields of a few Tesla, $a_o \sim 10^{-4} \text{ cm}$ while $a_B \sim 10^{-8} \text{ cm}$ so that $(a_B/a_o)^2 \simeq 10^{-8}$. Therefore, the phase factor γ is a negligibly small number or it can be said that no phase is gained. On the other hand, Arovas et al claim that there is a large gain. Since they have left out the variables, it is clear that they have over estimated the phase. The usual phase is ωt which is modified to $\omega t + \gamma$. If the area ma_o^2 is taken into account, γ tends to zero. If ω is the frequency and π/a_o is the wave vector, then the velocity is $\omega a_o / \pi$ and γ is not

quite independent of the velocity. If z_o is integrated in a clockwise sense around a circle of radius R , the values of $|z| < R$ contribute $2\pi i$ to the integral. Therefore,

$$\gamma_o = 2\pi\nu\phi/\phi_o. \quad (3)$$

Comparing this with (1),

$$\nu e = e^*. \quad (4)$$

Correcting the effective charge for the charge density,

$$\frac{e}{ma_o^2} = \frac{\nu e}{a_B^2} \quad (5)$$

we find,

$$\nu = \frac{a_B^2}{ma_o^2}. \quad (6)$$

If $a_B = a_o$, then $\nu = 1/m$. For $\phi = \phi_o$,

$$\gamma_o = 2\pi\nu \quad (7)$$

or the corrected value is

$$\gamma_o^{corr} = 2\pi\nu(a_B^2/a_o^2). \quad (8)$$

Since, $a_B \ll a_o$, γ_o^{corr} tends to zero. Hence, the phase pickup tends to zero. If there is a finite phase, then the vector potential \vec{A}_o should be corrected by adding to it a “statistical” vector potential A_ϕ such that,

$$\frac{e^*}{\hbar c} \oint \vec{A}_\phi \cdot d\vec{l} = \Delta\gamma = 2\pi\nu. \quad (9)$$

When above corrections are considered, this \vec{A}_ϕ tends to zero. As long as (7) is used, the phase for interchanging quasiparticles is $\Delta\gamma/2 = \pi$. That will change the sign for Fermi statistics. If ν is a fraction, then the quasiparticles cease to be interchanged like fermions so they obey the “fractional statistics”. We have lost contact with the anti-commutators but have not gained the commutators. Therefore, the particles are neither fermions nor bosons for certain fractional values of the phase. When A_ϕ tends to zero, we need not determine the statistics. When $\nu \rightarrow 0$, Arovas et al suggest the factor of ϕ_o for

bosons and $\phi_o(1 - 1/\nu)$ for fermions, but as $\nu \rightarrow 0$ there is a divergence in ϕ_o . Obviously, Wilczek and Zee³ prescription requires to be corrected. Some authors put the correction in the vector potential without correcting the scalar potential. Such a procedure can not give the correct “Chern-Simons” contribution. In the case of fermions, the interchange of quasiparticles produces a minus sign or a factor of -1, whereas in the case of bosons the required factor is +1. Therefore, the antisymmetry produces a factor of -1. In the case of product of fermion operators the factor produced will be ± 1 but never and never 3, then why Laughlin requires it to be 3? So, it will never be 3. Hence, Laughlin also requires to be modified. In the case of Peierls distortion, Su and Schrieffer⁴ suggest many manipulations of charge. In fact they divide the charge as $e/2$ and $e/2$ but no care is taken to divide the mass of the electron. When there is a Peierls distortion at twice the lattice constant, the wave vector becomes $\pi/2a$ and it is suggested that charge be counted as $e/2$ and $e/2$. It is more likely that charge will divide as 0 and e and not as $e/2$ and $e/2$. Similarly, if there are trimers, Su and Schrieffer⁴ suggest $e/3$ but here also no suggestion is given to divide the mass of the electron, so the charge may divide as 0, 0, and e and not as $e/3$, $e/3$ and $e/3$. It is obvious that Su and Schrieffer’s suggestions are at best qualitative and not found in real TTF-TCNQ or CH_x type compounds. It is true that Jackiw and Rebbi⁵ do find $\frac{1}{2}$ of a fermion number by Dirac equation but not by Peierls distortion.

3. Conclusions

The best one can say is that Su and Schrieffer’s suggestions are qualitative and not found in nature. Similarly, the ideas of fractional statistics are not set on good foundation. Even if the system is “incompressible” the results of Laughlin are not sufficiently rigorous and at best are arbitrary.

There has been a lot of drum beating in favour of composite fermions (CF), which have the flux quanta attached to the electron to modify the magnetic field but the flux attached electrons will be heavier than found in the experimental data by several orders of magnitude⁶. Similarly, the CF are much bigger objects than electrons to have the

same density. Hence the CF model is also not correct.

The correct theory of the quantum Hall effect is given in ref.7

6. References

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Note: Ref.7 is available from:

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